Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 2

25 October 2024

Zone A morning | **Zone B** morning | **Zone C** morning

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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A004

17 pages

8824–9701

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Consider the function $f(x) = 11\sqrt{x} - 2x - 11$, where $0 \le x \le 20$.

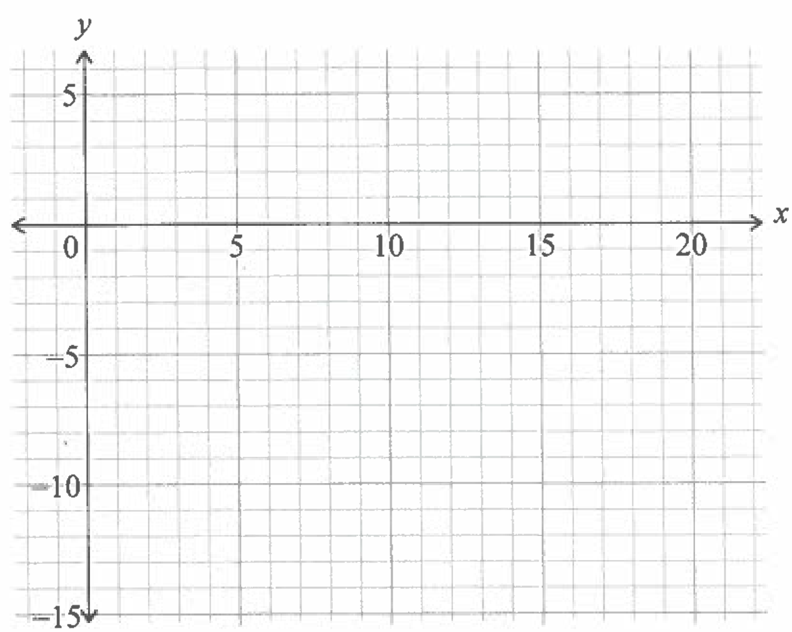
(a) Find the value of

(i) $f(0)$;

(ii) $f(20)$. [2]

(b) Find the two roots of $f(x) = 0$. [2]

(c) Sketch the graph of $y = f(x)$ on the following grid. [3]



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(Question 1 continued)

2. [Maximum mark: 4]

Find the coefficient of x^6 in the expansion of $(2x-5)^9$.

3. [Maximum mark: 5]

A discrete random variable, X, has the following probability distribution:

$$P(X = x) = \frac{kx}{20} \text{ for } x \in \{3, 5, 8, 11\}.$$

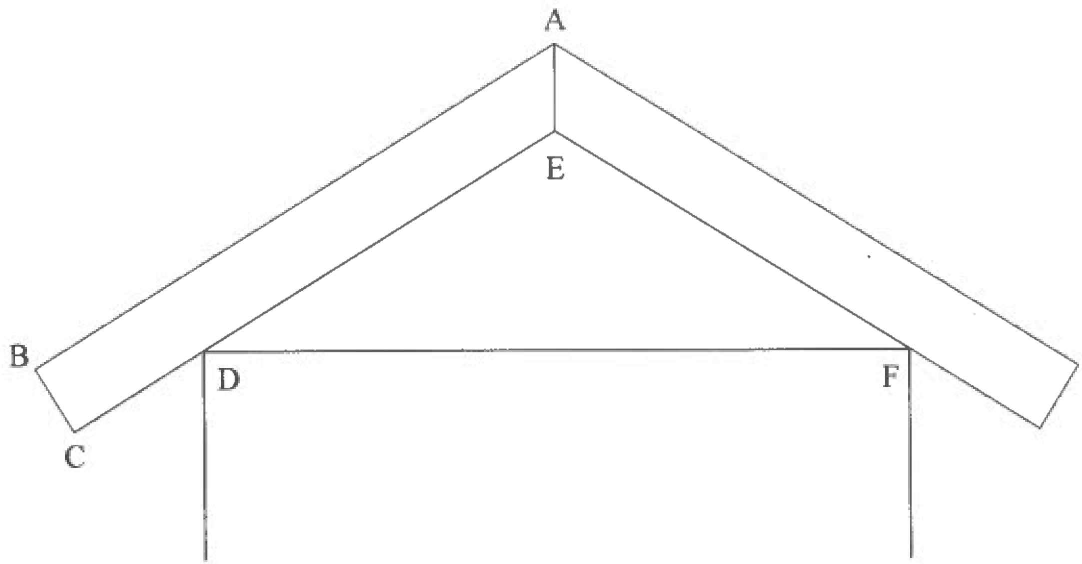
(a) Find the value of k. [2]

(b) Find $E(X)$. [3]

4. [Maximum mark: 8]

The following diagram shows the cross section of the roof of a house. The cross section is symmetrical about the vertical line through points A and E.

diagram not to scale



The gradient of [BA] is $\frac{7}{12}$.

(a) Find the size of $\mathrm{B\hat{A}E}$, expressing your answer in degrees. [3]

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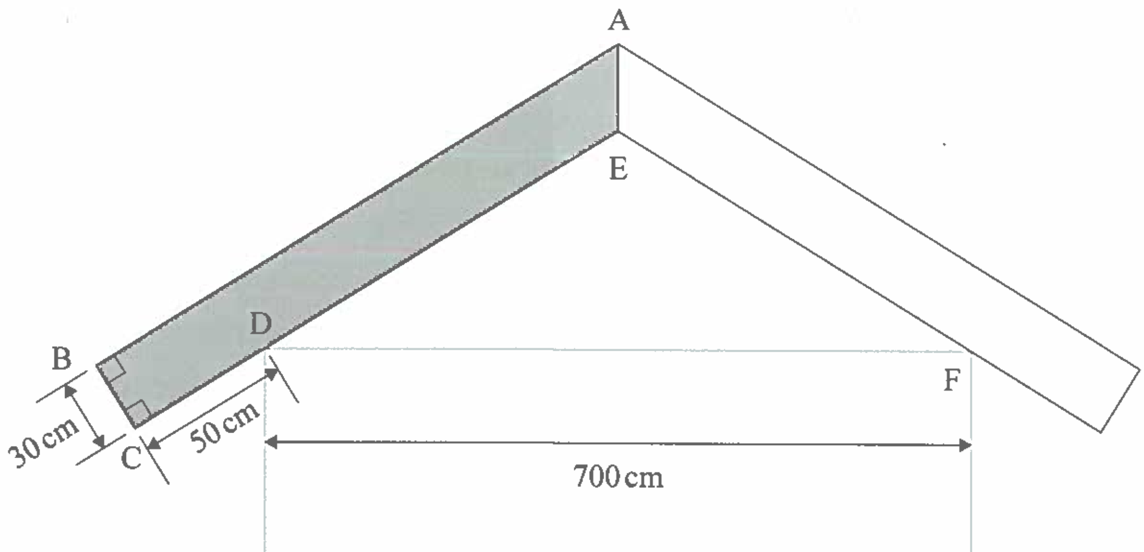
(Question 4 continued)

A builder requires the lengths of the sides [BA] and [CE].

The builder has the following measurements:

$A\hat{B}C = B\hat{C}E = 90°$, $DC = 50\,\text{cm}$, $BC = 30\,\text{cm}$, and $DF = 700\,\text{cm}$.

diagram not to scale



(b) Find

(i) CE;

(ii) BA. [5]

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5. [Maximum mark: 5]

Consider the function $h(x) = \log_{10}(4x^2 - rx + r - 1)$, where $x \in \mathbb{R}$.

Find the possible values of r. [5]

6. [Maximum mark: 8]

A continuous random variable X has probability density function

$$f(x)=\begin{cases}\frac{1}{5} & 0\le x<2\\ -\frac{x}{30}+\frac{4}{15} & 2\le x\le 8\\ 0 & \text{otherwise.}\end{cases}$$

(a) Find $\mathrm{E}(X)$. [3]

(b) Given that $\mathrm{E}(c-2X)=0$, where c is a constant, determine the value of c. [2]

(c) Find the median of X. [3]

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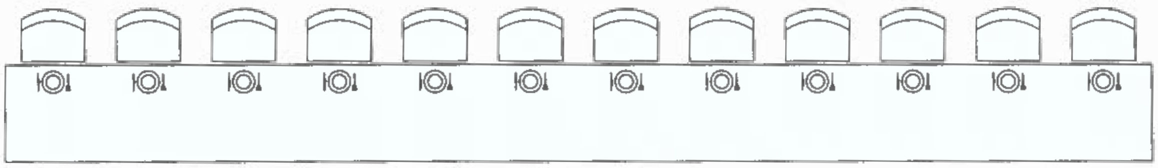
7. [Maximum mark: 6]

A self-service sushi restaurant has a row of 12 available seats, as shown in the following diagram.



Anvi, Vanya and Parita decide to go to the restaurant for lunch.

(a) Find the number of possible ways that they can be seated in this row, if they decide **not** to sit together as a group of 3. [3]

The next day, Anvi, Vanya and Parita are joined by 3 additional people in the same restaurant, and they sit in the same row of 12 available seats. Anvi, Vanya and Parita now decide to sit next to each other as a group of 3.

(b) Find the number of possible ways that these 6 people can be seated. [3]

8. [Maximum mark: 6]

(a) Given that $w \in \mathbb{C}$, prove that $ww^* = |w|^2$. [2]

Two complex numbers z and w satisfy the following equations:

$$5w^* = (1 - 2\text{i})z^2$$

$$zw = 10 + 10\text{i}.$$

(b) Given that $|w| = 2\sqrt{5}$, find z expressing your answer in the form $a + b\text{i}$, where $a, b \in \mathbb{Z}$. [4]

9. [Maximum mark: 7]

The sum of the first n terms of a geometric sequence is given by $S_n = \sum_{r=0}^{n-1} 5(\log_2 c)^r$.

(a) Given that S_n converges, find the range of possible values of c. [3]

(b) In the case where $c = 1.5$, find the least value of n such that $|S_\infty - S_n| < 0.1$. [4]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 15]

The following table shows the population of Canada t years after the year 2000.

t (years after 2000)	0	5	10	15	20
p (population in millions)	30.7	32.2	34.0	35.7	37.9

A student uses linear regression to model the population of Canada using these data. The student model is $p = at + b$.

(a) (i) Write down the value of a and the value of b.

(ii) Interpret, in context, the value of a. [3]

The student uses this model to predict the population of Canada in the year 2030, where $t = 30$, and calculates a population of approximately 41.3 million people.

(b) Comment on the reliability of the student's prediction. [1]

A data scientist, Benoit, uses additional information to develop an exponential model for Canada's future population.

In this model, $B(t) = 33.5(1.005)^t$ represents the millions of people in Canada t years after the year 2000, where $25 \leq t \leq 100$.

(c) (i) Use Benoit's model to predict the population of Canada in the year 2100.

(ii) Interpret, in context, the value 1.005 in Benoit's model. [3]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 10 continued)

Another data scientist, Cecilia, develops a third model for the Canadian population.

In this model, $C(t) = \frac{62}{1 + e^{-0.02t}}$ represents the millions of people in Canada t years after the year 2000, where $25 \le t \le 100$.

(d) Use Cecilia's model to predict the population of Canada in the year 2100. [1]

(e) Determine the year in which the difference between the predictions from Benoit's model and Cecilia's model is greatest. [3]

(f) Find the value of

(i) $B'(75)$;

(ii) $C'(75)$. [2]

(g) Compare and interpret, in context, the values of $B'(75)$ and $C'(75)$. [2]

Do **not** write solutions on this page.

11. [Maximum mark: 18]

A line L is defined by $L: \ -\frac{x}{2}+1 = y+4 = \frac{z}{3}$.

(a) Find the equation of L, expressing your answer in the form $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$, where $\lambda \in \mathbb{R}$. [3]

(b) Determine the minimum distance from the origin O to the line L. [5]

A plane Π is defined by $\Pi: \ 6x - 3y + 5z = 24$.

(c) Verify that Π contains L. [3]

A second line M is parallel to Π.

The line M passes through the point $(4, 1, 2)$ and intersects the z-axis.

(d) Find the equation of M, expressing your answer in the form $\boldsymbol{s} = \boldsymbol{c} + \mu\boldsymbol{d}$, where $\mu \in \mathbb{R}$. [7]

Do **not** write solutions on this page.

12. [Maximum mark: 21]

A curve C has equation $y = \frac{2x^2+6x-3}{x+k}$, $x \in \mathbb{R}$, $x \neq -k$, where k is a real positive constant.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x^2+4kx+6k+3}{(x+k)^2}$. [4]

(b) Find the smallest value of k for which a local minimum or maximum point exists. [4]

Consider the curve C, when $k=2$.

(c) Write down the equation of the vertical asymptote. [1]

(d) Find the equation of the oblique asymptote. [4]

(e) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} > 2$, for $x \in \mathbb{R}$, $x \neq -2$. [4]

(f) Sketch the curve C, showing clearly both asymptotes and the general behaviour of C as it approaches each asymptote. [You are not required to find any axes intercepts.] [4]